METHODS OF CALCULATING RADIATIVE—CONDUCTIVE HEAT EXCHANGE IN THE COMPONENTS OF ELECTROVACUUM INSTRUMENTS

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Two methods are proposed for solving the equations of radiative—conductive heat exchange — analytic and computational. The efficiency of both methods is illustrated by specific examples of a calculation of the temperature fields and radiation fluxes at the cathode.

Electrovacuum instruments (EVI) are widely used in different fields of technology. The reliability and durability of their operation depends to a strong degree on thermal factors, the analysis of which is a complex mathematical problem. At present, because of the sharp increase of the requirements on the reliability and longevity of EVI, the problem concerning the development of efficient engineering methods of calculating their thermal cycle acquires great practical importance. Taking this requirement into consideration, the present paper explains certain methods of calculating the heat exchange in the components of EVI, the use of which will permit the necessary information about the thermal state of the instrument to be obtained quite rapidly.

Formulation of the Problem. From the thermophysical point of view, an electrovacuum instrument is a system of n components, located in a vacuum and exchanging thermal energy between themselves. The transfer of thermal energy from one element to another can be accomplished by two methods: by thermal conductivity and by radiation.

It is well known that the calculation of radiative—conductive heat exchange in such systems is associated with the solution of a number of problems, of which the most complex are the following: 1) nonlinearity of the starting system of equations; 2) the presence of a large number of components mutually irradiating one another and in contact with one another; 3) the complexity of the geometric shape of some of them (spirals, networks of different form, gaps of arbitrary geometry, etc.). Because of these difficulties, an exact calculation of the temperature fields and radiation fluxes in these systems is possible only by numerical methods. However, when the number of components is large ($n > 5$), the use of direct numerical methods becomes ineffective, because of their unwieldiness. In these cases, it is advantageous to use semianalytical methods of calculation, the essence of which consists in the following.

With specified thermophysical parameters, the nature of the temperature distribution in the i-th component participating in the heat exchange still with $(n - 1)$ components depends on the values of the temperature gradients T_{ij}, the distribution of internal heat sources, the density distribution of the incident radiation flux $E_{inc.i}$ and the intensity of the inherent radiation of the component $\sigma_0\varepsilon_i T_i^4$. In those cases when $|\lambda_i \mathrm{grad}\, T_i| > |\sigma_0\varepsilon_i T_i^4 - \varepsilon_i E_{inc.i}|$, and for the metallic components of the EVI this condition, as a rule, is satisfied, it is advantageous to use a method of calculation based on the approximate evaluation of $E_{inc.i}$ and the boundary temperatures of the components T_{ij} not of a rigorous system of n integral and n differential equations of radiation and thermal conductivity (which is almost impossible to solve), and from its approximating system in which all the equations, except two referring to the i-th component, are replaced by algebraic equations. The errors resulting from this does not strongly distort the temperature distribution in the i-th component, if only the inequality stated above is satisfied.

The use of average values of the integrated radiation fluxes, occurring as variables in the required algebraic equations, strongly simplify the solution of the problem with respect

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to the temperature distribution in the i-th component, as in this case the necessity for solving the system of n integral radiation equations and n differential thermal conductivity equations is eliminated. The next two systems in this case decay into a system consisting of one integral and one differential equation, and into a system consisting of 2(n − 1) algebraic equations with unknowns $\overline{Q}_{inc.k}$ and $\overline{T}_k$ (k = 1, 2, ..., n − 1), where $\overline{Q}_{inc.k}$ is the average value of the integrated radiation flux, incident on the surface of the k-th component and $\overline{T}_k$ is the average temperature of the surface of this component.

The system, consisting of one integral and one differential equation, determining the temperature distribution and the density of the radiation fluxes in the i-th component, can be solved by one of the approximate analytical methods — by the method of perturbations. As the perturbing parameter is chosen, the degree of blackness of the component with respect to which the temperature distribution is being determined. As the majority of the components of electrovacuum instruments are made of metals or their alloys, the degree of blackness of which is less than 0.3, then this method allows a quite accurate solution to be obtained, even in the first approximation.

In practice, the scheme of the solution described may be complicated, because the necessity arises to determine the temperature distribution for two or more components. In this case, the general scheme of solution of these problems remains the same, but it is necessary to find the joint solution of two further differential and two integral equations, i.e., the complexity of the solution increases strongly. Usually, from the analysis of the physical pattern of the distributions of the sources and heat sinks in the system of n components, it can always be established for which components the temperature distribution should be determined, and for which it should not. Consequently, the method of calculation of the temperature fields and radiation fluxes described cannot be applied formally. It requires a preliminary analysis of the distribution pattern of the sources and heat sinks in the system of n components.

Let us illustrate the use of the method described by the example of a calculation of the thermal cycle of an oxide cathode.

Solution of the Radiative-Conductive Heat-Exchange Equations for an Oxide Cathode by the Method of Perturbations. Figure 1 schematically shows the oxide cathode. It consists of six basic components. Component 1 is the oxide coating of the cathode. The oxide is a strongly porous medium, consisting of individual grains with a size of 1-3 μm. The grains, representing a mixture of BaO, CaO, and SrO crystals, are semitransparent to thermal radiation. Therefore, the oxide can be considered as a gray medium, weakly absorbing and strongly scattering radiation. The thickness of the layer is 50-100 μm. Because of the strong porosity, the effective value of the coefficient of thermal conductivity of the oxide is very small (on the order of 10^{-4} W/cm•deg). Because of this, large temperature drops over the oxide are possible. In connection with this, the main problem in calculating the thermal cycle of the oxide cathode is to determine the temperature of the oxide surface, on which the emission current of the cathode strongly depends [1].

Component 2 consists of two items made of nickel — the core of the cathode and the chamber for the heater — forming a closed cavity. The wall thickness of the cavity is 0.03-0.05 cm. Inside the cavity is located a wire heater of bifilar construction (component 3). The thickness of the wire is 100-200 μm and it has a length of tens of centimeters. The cathode support (component 4) is made of alloys with a low value of the coefficient of thermal conductivity. The wall thickness of the support varies in the limits 0.005-0.1 cm, and its length is of order 1 cm. Component 5 is a screen for reducing radiation losses from the cathode and is made of nickel with a thickness of order 0.05 cm. The cathode is braced to the foot of the electrovacuum instrument (component 8) by means of metal strips (component 6), as a rule made of nickel with a thickness of order 0.05 cm. The cathode is located in a vacuum tank (component 7).

The scheme for calculating the thermal cycle of this cathode includes the problem of solving the system of heat-exchange equations, describing the process of heat transfer by radiation and thermal conductivity [2-4]. This system has the form:

$$\lambda_1 \frac{d^2T_1}{dz^2} - 4\sigma_0 n^2 \alpha T_1^4(z) + \alpha G(z) = 0, \quad 0 < z < l_1, \tag{1}$$

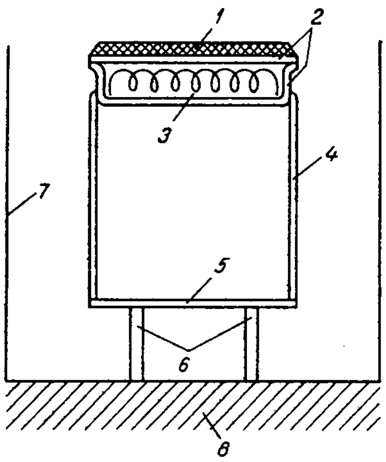

Fig. 1. Schematic diagram of the oxide cathode.

$$G(z) = 2\Phi_1 E_2(z) + 2\Phi_2 E_2(l_1 - z) + 2\sigma_0 n^2 \alpha \int\limits_0^{l_1} T_1^4(x) E_1(|z - x|)\, dx \tag{2}$$

$$+ \frac{1}{2} \gamma \int\limits_0^{l_1} G(x) E_1(|z - x|)\, dx,$$

$$U_h I_h + \lambda_4 \left(\frac{dT_4}{dz}\right) \pi (R_4'^2 - R_4^2) + \varepsilon_2 Q_{inc,2} + \lambda_1 \left(\frac{dT_1}{dz}\right)_{z=0} \pi R_1^2 - \sigma_0 n^2 \varepsilon_2 \overline{T}_2^4 \pi R_4^2 + q_1 = 0, \tag{3}$$

$$U_h I_h = 2\pi (R_3 + \delta_3)\, l_3 \sigma_0 \varepsilon_{pr} (\overline{T}_3^4 - \overline{T}_2^4), \tag{4}$$

$$\overline{T}_3 = \frac{U_h \pi R_3^2}{I_h a l_3} - \frac{b}{a}, \tag{5}$$

$$\lambda_4 \frac{d^2 T_4}{dz^2} + \frac{2\varepsilon_4 R_4}{(R_4'^2 - R_4^2)} E_{inc,4}(z) = \frac{2\sigma_0 (\varepsilon_4' R_4' + \varepsilon_4 R_4)}{(R_4'^2 - R_4^2)} T_4^4(z), \quad 0 < z < l_4, \tag{6}$$

$$E_{inc,4}(z) = [\sigma_0 \varepsilon_2 \pi R_4^2 \overline{T}_2^4 + (1 - \varepsilon_2) \overline{Q}_{inc,2}] \frac{\varphi_{24}(z)}{2\pi^3 R_4^2} + \frac{R_4}{2\pi^2} \int\limits_0^z [\sigma_0 \varepsilon_4 T_4^4(x) + (1 - \varepsilon_4) E_{inc,4}(x)]\, \varphi_{4'4}(x, z)\, dx$$

$$+ \frac{R_4}{2\pi^2} \int\limits_z^{l_4} [\sigma_0 \varepsilon_4 T_4^4(x) + (1 - \varepsilon_4) E_{inc,4}(x)]\, \varphi_{44'}(x, z)\, dx, \tag{7}$$

$$\overline{Q}_{inc,2} = \overline{\psi}_{52} [\sigma_0 \varepsilon_5 \pi R_4^2 T_5^4 + (1 - \varepsilon_5) Q_{inc,5}] + \frac{R_4}{\pi} \int\limits_0^{l_4} [\sigma_0 \varepsilon_4 T_4^4(x) + (1 - \varepsilon_4) E_{inc,4}(x)]\, \varphi_{42}(x)\, dx, \tag{8}$$

$$\overline{Q}_{inc,5} = \overline{\psi}_{25} [\sigma_0 \varepsilon_2 \overline{T}_2^4 + (1 - \varepsilon_2) \overline{Q}_{inc,2}] + \frac{R_4}{\pi} \int\limits_0 [\sigma_0 \varepsilon_4 T_4^4(x) + (1 - \varepsilon_4) E_{inc,4}(x)]\, \varphi_{45}(x)\, dx, \tag{9}$$

$$-\lambda_4 \left(\frac{dT_4}{dz}\right)_{z=l_4} \pi (R_4'^2 - R_4^2) + \varepsilon_5 \overline{Q}_{inc,5} = 2\sigma_0 \varepsilon_5 \pi R_4^2 \overline{T}_5^4 + \lambda_6 \frac{T_5 - T_0}{l_6} \Delta S_6. \tag{10}$$

The system of equations (1)-(10) must be supplemented by the boundary conditions for the functions $T_1(z)$ and $T_4(z)$, which have the simple form:

$$\left(\frac{dT_1}{dz}\right)_{z=l_1} = 0; \quad T_1(0) = \overline{T}_2; \; T_4(0) = \overline{T}_2; \quad T_4(l_4) = \overline{T}_5. \tag{11}$$

The quantities Φ_1, Φ_2, and q_1 are calculated by the formulas:

$$\left.\begin{aligned}\Phi_1 &= \frac{1}{\Delta_1}\,[L_1 + 2(1-\varepsilon_2)E_3(l_1)L_2],\\ \Phi_2 &= \frac{1}{\Delta_1}\,[2\rho_0 E_3(l_1)L_1 + L_2],\end{aligned}\right\} \tag{12}$$

$$q_1 = \frac{1}{\Delta_1}\cdot 4\rho_0\varepsilon_2^2\pi R_1^2\sigma_0 n^2 E_3^2(l_1) + \pi R_1^2\varepsilon_2\int_0^{l_1}\left[\alpha\sigma_0 n^2 T_1^4(x) + \frac{\gamma}{4}G(x)\right]K(x,\ l_1)\,dx, \tag{13}$$

where

$$\Delta_1 = 1 - 4\rho_0(1-\varepsilon_2)E_3^2(l_1), \tag{14}$$

$$L_1 = \sigma_0 n^2\varepsilon_2\overline{T}_2^4 + 2(1-\varepsilon_2)\sigma_0 n^2\alpha\int_0^{l_1}T_1^4(x)E_2(x)\,dx + \frac{1}{2}(1-\varepsilon_2)\gamma\int_0^{l_1}G(x)E_2(x)\,dx, \tag{15}$$

$$L_2 = 2\sigma_0 n^2\rho_0\alpha\int_0^{l_1}T_1^4(x)E_2(l_1-x)\,dx + \frac{1}{2}\gamma\rho_0\int_0^{l_1}G(x)E_2(l_1-x)\,dx, \tag{16}$$

$$K(x,\ l_1) = \frac{1}{\Delta_1}\,4\rho_0 E_3(l_1)E_2(l_1-x) + 2E_2(x)\left[1 - \frac{4\rho_0(1-\varepsilon_2)E_3^2(l_1)}{\Delta_1}\right]. \tag{17}$$

The system of equations (1)-(10) must be solved relative to I_h, $\overline{T}_3$, $\overline{T}_2$, $\overline{T}_5$, $T_1(z)$, $G(x)$, $T_4(z)$, $\overline{Q}_{inc.2}$, $\overline{Q}_{inc.5}$, and $E_{inc.4}(z)$.

The solution of the nonlinear system of equations (1)-(10) with the boundary conditions (11) will be worked out by the method of perturbations. Following the formalism of this method, we expand all the unknown in series in powers of α and ε_4:

$$X_i \approx X_i^{(0)} + \left(\frac{\partial X_i}{\partial\alpha}\right)_{\alpha=\varepsilon_4=0}\alpha + \left(\frac{\partial X_i}{\partial\varepsilon_4}\right)_{\alpha=\varepsilon_4=0}\varepsilon_4\ (i = 1,\ 2,\ \ldots,\ 10), \tag{18}$$

where $X_i^{(o)}$ is the solution of the system of equations (1)-(10) for $\alpha = 0$ and $\varepsilon_4 = 0$.

If we put α and ε_4 equal to zero in Eq. (1)-(10), then we obtain a linear system of equations relative to the functions $T_1^{(o)}(z)$, $T_4^{(o)}(z)$, $E_{inc.4}^{(o)}(z)$, and $G^{(o)}(z)$, the solution of which does not cause difficulties. In this case, system (1)-(10) reduces to a system of nonlinear algebraic equations for calculating the average values of $\overline{Q}_{inc.i}$ and $\overline{T}_i$. The latter is elementary solved numerically by the standard program. Having determined $X_i^{(o)}$ in this way, we differentiate Eq. (1)-(11) with respect to α and, assuming then that α and ε_4 are equal to zero, we obtain a linear system of equations for determining the ten unknowns $(\partial X_i/\partial\alpha)_{\alpha=\varepsilon_4=o}$. Then we differentiate system (1)-(11) with respect to ε_4, and, assuming that ε_4 and α are equal to zero, we again obtain a linear system of equations for determining the ten unknowns $(\partial X_i/\partial\varepsilon)_{\alpha=\varepsilon_4=o}$. Thus, the method of perturbations converts one nonlinear system of heat-exchange equations into three linear systems, for which the analytical methods of solution are well developed and will cause no difficulties.

The accuracy of the method of perturbations can be judged by the curves shown in Fig. 2 and 3, where 1 are the results of the calculations of the temperature distribution and the incident radiation flux density for component 4 (Fig. 1), obtained by the method of perturbations; 2 are the similar curves, obtained by means of solving the systems of equations (6) and (7) by a direct numerical method, described in the next section.

Solution of the Heat-Exchange Problem in a Cathode Unit by the Numerical Method. As already pointed out, an exact calculation of the temperature fields and radiation fluxes is possible only on the basis of the use of numerical methods. Among them, the most promising is the finite-element method [5]. We shall show the special features of the use of this method by an example of the calculation of the temperature field of the cathode unit support (4 in Fig. 1).

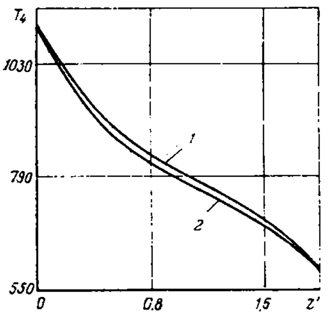

Fig. 2

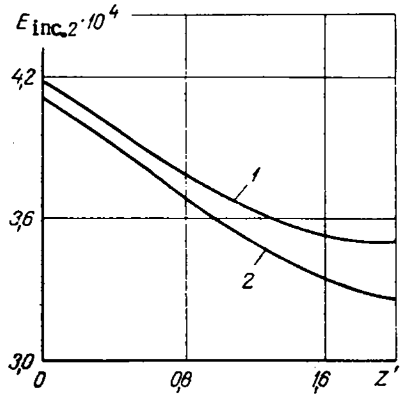

Fig. 3

Fig. 2. Temperature distribution T_4, °K, over the height of the cathode support $z' = z/R_4$: 1) calculation by the method of perturbations; 2) by the numerical method.

Fig. 3. Density distribution of incident radiation flux $E_{inc.4}$, W/m², over the height of the cathode support $z' = z/R_4$: 1) calculation by the method of perturbations; 2) by the numerical method.

Just as previously, we shall neglect the temperature drop over the wall thickness of the support, which corresponds to the use of one-dimensional finite elements. In solving the problem, the reference region is divided up arbitrarily into individual sections — cylindrical shells of finite height — and within the limits of each of them, the temperature distribution $T_4(z)$ is approximated by a quadratic parabola. Its coefficients are related unambiguously with the values of the required temperature at three nodal points of each section, and the problem reduces in this way to the calculation of the nodal temperatures.

The system of equations for determining these temperatures is obtained from the condition of stationarity of the functional, equivalent to the boundary problem, including in addition to the differential equation (6) the boundary conditions (11). It is well known that this functional has the form

$$I[T_4] = \int_0^{l_4} \left\{ \lambda_4 \frac{dT_4(z)}{dz} + 4\,\frac{(\varepsilon_4' R_4' + \varepsilon_4 R_4)}{(R_4'^2 - R_4^2)}\, T_4^5(z) - \frac{4\varepsilon_4 R_4}{(R_4'^2 - R_4^2)}\, T_4(z)\, E_{inc\,4}(z) \right\} dz. \quad (19)$$

If, in Eq. (19), we substitute the approximations assumed for $T_4(z)$ within the limits of each section, then it is found that the functional $I[T_4]$ degenerates into a function of N nodal values of temperature T_{4i}. Writing the necessary conditions of the extremum of this function, we obtain a system of N algebraic equations relative to the nodal temperatures, of the following form:

$$A\mathbf{T}_4 = B\overline{\mathbf{E}}_{inc} - C\overline{\mathbf{T}}_4^4\,. \quad (20)$$

Introducing for $E_{inc.4}(z)$ the stepwise approximation, which usually is assumed in the zonal method [2], instead of Eqs. (7)-(10) we also obtain a system of algebraic equations

$$R\overline{\mathbf{E}}_{inc} = F\overline{\mathbf{T}}_4^4\,. \quad (21)$$

Systems (20) and (21) obtained are solved jointly by the iteration method according to the following scheme:

$$A\mathbf{T}_{4(k+1)} = B\overline{\mathbf{E}}_{inc\cdot(k)} - C\overline{\mathbf{T}}_{4(k)}^4,\; R\overline{\mathbf{E}}_{inc\cdot(k+1)} = F\overline{\mathbf{T}}_{4(k+1)}^4\,, \quad (22)$$

where k is the iteration number, so that when k = 0, it is supposed that $\overline{T}_{40}^4 = 0$ and $\overline{E}_{inc.0} = 0$. The method solution described is easily programmed and converges quite rapidly.

NOTATION

T_0, foot temperature $\overline{T}_3$, average temperature of heater; $\overline{T}_2$ and $\overline{T}_5$, mean temperatures of components 2 and 5; $T_1(z)$ and $T_4(z)$, functions of the temperature distribution over the thickness of the oxide and over the height of the support 4; $\alpha G(x)$, the quantity of radiant energy absorbed by unit volume of oxide; $\overline{Q}_{inc.i}$ (i = 2.5), mean values of the integrated radiation fluxes incident on the surface of components 2 and 5; $E_{inc.4}(z)$, radiation flux density, incident on the surface of component 4; I_h and U_h, the current and voltage of the heater filament; a and b, coefficients defining the linear dependence $a\overline{T}_3 + b$ of the specific electrical resistance of the heater on the temperature $\overline{T}_3$; δ_3, thickness of heater insulation; ε_i, λ_i, and l_i, the degree of blackness, coefficient of thermal conductivity, and linear size of the element with number i; α and γ, mean values of the coefficients of absorption and scattering of radiation by the oxide; n, refractive index of oxide; ρ, coefficient of reflection of radiation at the oxide—vacuum boundary; ψ_{ij}, mean values of the integrated angular coefficients of emission of the surfaces of the components; $\varphi_{ij}(z)$, geometric coefficients of emission of a cylindrical element of height dz; $E_n(z)$, exponential integrals [4]; R_1, radius of the cathode; R_3, radius of the heater wire; R_4' and R_4, outside and inside radii of the cylindrical support 4; ΔS_6, cross-sectional area of the mounting strips; T_4, unknown vector of the nodal values of the temperature of the cathode support; T_4^4, vector of the fourth power of the values of the required temperatures within the limits of the reference sections; $E_{inc.}$, vector of the mean values of the incident flux densities on the inside surface of the sections of the support 4 and screens 2 and 5; A, B, C, R, and F, matrices of the coefficients of the systems of equations (20) and (21).

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